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Reply

# Folding with thermal mechanical feedback: Another reply

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ABSTRACT

In response to Schmid et al. (2010): (i) The linear Biot theory assumes fold wavelengths grow independently of each other; this is the "Biot process". (ii) The Biot theory predicts that only one wavelength grows to finite amplitudes; a spread of wavelengths at finite amplitudes indicates non-Biot processes operate. (iii) Boundary conditions control the wavelength that grows. (iv) Non-linear behaviour can result in non-Biot behaviour such as localised folding with no dominant wavelength. (v) Strain-rate softening is one form of non-linearity and leads to folding and boudinage at all scales; thermal–mechanical feedback leads to strain-rate softening producing folding and boudinage at the kilometre scale. (vi) The larger the viscosity ratio the larger the feedback effect. (vii) The Biot process may be important in some deformed rocks but others perhaps dominate.



## 1. What is and is not part of the Biot theory?

Equations describing the finite deformation of a layer or series of layers are discussed by Hunt et al. (1997) and Muhlhaus et al. (1994, 1998). In the general case these equations are partial differential equations, fourth order in spatial variables but also a function of time and may or may not be solvable using Fourier expansion methods. If the constitutive equations are linear and the deflections are small then many of the complicated terms (in particular the time derivatives) vanish and one arrives at the classical Biot expression. These governing equations can still represent elastic, viscous or plastic behaviour, as Biot (1965) points out, so the theory is powerful. We have the highest respect for Biot's work and have never implied that this is at fault or should be neglected. The essence of Biot's theory is that the deformation of layered materials is intrinsically unstable and folds begin to grow depending on the mechanical contrast between layers. The analysis assumes (true for linear systems and small deflections) that the wavelengths that begin to grow can be represented by a Fourier series and that the growth of each wavelength is independent of all others. The emphasis is on discovering which wavelength grows fastest and this becomes the dominant wavelength. The analysis is strictly applicable *only* to the moment the instabilities begin to grow and only for certain linear systems can one extrapolate this result to finite amplitudes. In general the growth of initial perturbations to finite size cannot be predicted from the results of a linear perturbation analysis. It is possible, for instance, that other instabilities can develop sequentially after the first instability (Hunt et al., 2006). Extensions of the small deflection theory to large deflections (Muhlhaus et al., 1994, 1998) are still part of the Biot approach so long as growth-independence of the unstable modes is preserved. If non-linear behaviour is included then the assumption of growth-independence may not be true and approaches other than the linear Fourier analysis need to be implemented.

There are now two questions to be asked of the strict small amplitude Biot theory: (i) What happens at large deflections? (ii) What happens if the system is non-linear as arises from geometrical softening, from strain softening or from strain-rate softening?

### 1.1. Finite amplitude folding

As far as we are aware, the only strictly analytical solutions to large amplitude viscous folding are those by Muhlhaus et al. (1994, 1998) which show that the Biot theory at small deflections can be extended to large deflections but *only one wavelength grows to large amplitudes* and this wavelength is governed by the boundary conditions. This result is supported by all computer models we know of (including those reported in Schmid et al., 2010). Thus the assertion by Schmid et al. (2010) that a spectrum of wavelengths is to be expected, at large amplitudes, from the Biot theory is at odds with the analytical solutions and assumes that the dispersion of wavelengths at the moment of instability is reflected in the subsequent finite amplitude folds. The analytical solutions show that the dominant wavelength grows preferentially, is the only one preserved after relatively small strains and depends on the boundary conditions as we discussed in Hobbs et al. (2008). We take the data that show a spread of wavelength to thickness ratios in natural folds to indicate that processes other than the strict Biot process are operating in nature.

### 1.2. Influence of non-linearities

Hunt et al. (1997) emphasise that the Biot theory is a special case of a general approach to folding in which non-linearities are

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relevant. The general outcome of such considerations is that the wavelengths that grow are not independent of each other, with interference between modes, and the result can be localisation of the folding behaviour; *the concept of a dominant wavelength need no longer exist* and complicated wave packets can form (Hunt and Wadee, 1991). This behaviour is beyond the reach of linear theories such as those considered by Biot. A vast literature on non-linear bifurcation theory has appeared since Biot's important 1965 book allowing one to analyse some forms of non-linear behaviour. One way of establishing the nature of such non-linear behaviour is through modern continuum thermodynamics which we started to explore in Hobbs et al. (2008, in press) and Regenauer-Lieb et al. (2009).

An important example of non-Biot folding is the development of kink, chevron and concentric folds in a multilayer stack of thin elastic layers (Wadee et al., 2004; Edmunds et al., 2005) where the non-linearity stems from geometrical softening associated with large rotations. The critical load required to initiate instabilities is modelled using the Maxwell stability criterion; folding initiates *after* the stored elastic energy matches the energy required for slip on the layers. Such behaviour is non-linear and relies on bifurcation theory to define the initial and subsequent initiation and growth of instabilities. The concept of a dominant wavelength is meaningless here. The difference between this and the Biot approach is that Biot (1965, p204; and others) regarded the Maxwell stability criterion as defining the initiation of kink folds in anisotropic materials whereas bifurcation theory regards instability as developing *after* the energy minimum is passed. The Biot theory gives no information on when folds nucleate and how the system evolves.

Hobbs et al. (2008) present another example of non-linear, non-Biot behaviour during buckling. The constitutive behaviour of a temperature dependent Newtonian or power-law viscous material, with thermal–mechanical feedback, is modified such that the effective viscosity becomes a decreasing function of strain-rate. For a Newtonian material, the effective viscosity decreases with the square of the strain-rate (Fleitout and Froidevaux, 1980). Similar strain-rate softening relationships arise from other feedback processes including mineral reaction-mechanical coupling (Hobbs et al., in press) and diffusion-mechanical feedback (Regenauer-Lieb et al., 2009). Strain-rate perturbations are amplified during deformation leading to greater localised strain-rates and self-enhancing feedback.

The general theory for instabilities in rate dependent materials with strain and strain-rate softening is presented in many papers; examples are Anand et al. (1987) and Needleman (1988). One conclusion is that instability is sensitive to geometrical or physical heterogeneities. With reference to folding, heterogeneities in strain-rate introduced by incipient buckling of a layer are amplified by strain-rate softening. These heterogeneities may be those predicted by the Biot theory but from the moment instabilities grow, the process is not one of Biot-type wavelength selection but involves localisation of deformation. In Hobbs et al. (2008) we selected constitutive relations for quartz and feldspar that give realistic but small (<20) viscosity ratios between layers under mid- to lower-crustal conditions. This selection is based on careful and critical analysis by Hirth et al. (2001). Of course other constitutive relations exist but as Schmid et al. (2010) point out they can give very large viscosity ratios, so large in fact that one has to question the validity of extrapolating the experimental data to geological conditions. A viscosity ratio of $10^3$ at 1000 K in a homogeneously shortening layered material implies magnitudes of differential stress in the crust of Gigapascals if the strength of weak layers is as low as 1 MPa. Such high values seem unlikely. An additional issue is that the higher the viscosity ratio, the higher the viscous dissipation in the competent layer(s) leading to thermal–mechanical feedback effects even more dramatic than are considered by Hobbs et al. (2008). In the presence of strain-rate softening feedback, the larger the initial viscosity ratio the more intense the fold localisation. Fig. 1 shows shortening of a single layer with a relatively high (200) viscosity ratio to the embedding matrix. With no strain-rate softening (Fig. 1a and b) the Biot result of a sinusoidal wave form develops; with strain-rate softening (Fig. 1c and d) localised folding develops. The effects discussed in Hobbs et al. (2008) are conservative compared to what is expected at larger viscosity ratios and tend to support conclusions that these feedback effects are important in natural deformations rather than detract from them.

## 2. Influence of boundary conditions

One outcome from the vast literature (for instance Shawki, 1986; Fressengeas and Molinari, 1987) concerning the influence of boundary conditions on deformation is that if the boundary conditions comprise constant imposed velocity then the acceleration of the boundaries is zero and the force within a viscous material must decay with time. This is obvious from Newton's Second Law of Motion. The outcome (Muhlhaus et al., 1994, 1998) for a viscous material is that the amplification of deflections in the layer must decrease with time. Ultimately buckling ceases even though shortening continues; the only deformation is homogeneous amplification of early formed deflections. Schmid et al. (2010) understandably neglect these analyses because it means that the

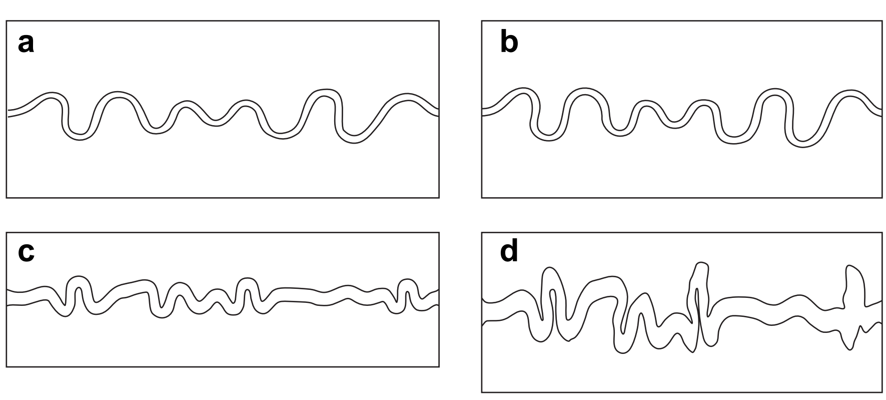


**Fig. 1.** Results of shortening a single layer without (a), (b) and with (c), (d) strain-rate softening. Constant velocity boundary conditions. Maxwell materials in both cases. Elasticity homogeneous throughout. Initial viscosity ratio 200. For details of constitutive relations see Hobbs et al. (in press). The size of these folds is independent of a length scale but once the process operating to produce strain-rate softening in (c) and (d) is specified a length scale is set (see Table 1). 44% Shortening in (a) and (c), 50% shortening in (b) and (d).

**Table 1**
List of processes and typical length scales at which that process dominates.

| Process | Diffusivity, $m^2 s^{-1}$ | Strain-rate, $s^{-1}$ | Length scale for process, m | Reference |
|---|---|---|---|---|
| Heat conduction; slow deformations (tectonic deformations) | $10^{-6}$ | $10^{-12}$ | 1000 | Hobbs et al. (2008) |
| Heat conduction; fast deformations (slow to fast seismic) | $10^{-6}$ | $10^{-2}$–$10^{2}$ | $10^{-2}$–$10^{-4}$ | Veveakis et al. (in press) |
| Chemical diffusion; slow deformations (tectonic deformations) | Say $10^{-10}$–$10^{-16}$ | $10^{-12}$ | 10–$10^{-2}$ | Regenauer-Lieb et al. (2009) |
| Chemical diffusion; fast deformations (slow to fast seismic) | Say $10^{-10}$–$10^{-16}$ | $10^{-2}$–$10^{2}$ | $10^{-4}$–$10^{-7}$ | Veveakis et al. (in press) |
| Fluid diffusion | Depends on permeability | $10^{-12}$–$10^{-2}$ | Any value from 1000 to $10^{-4}$ depending on permeability | |
| Chemical reactions | No diffusivity. Coupling depends on chemical dissipation | | All scales from 1000 to $10^{-4}$ | Hobbs et al. (2009); Regenauer-Lieb et al. (2009); Veveakis et al. (in press) |

dominant wavelength in a natural fold system cannot be used to say anything about the viscosity ratio unless one knows the boundary conditions. Although the arc length may be important in discussions of wavelength to thickness ratios for constant force boundary conditions, for constant velocity boundary conditions, thickening occurs late in the folding history (Figure 3 of Hobbs et al., 2008) and hence the arc length is not the relevant length scale.

The results of Schmid et al. (2010; Figure 3) illustrate this point perfectly; they show that after 50% shortening the fold amplitudes, for small viscosity ratios, are very small. This is entirely due to boundary conditions of constant velocity which result in low amplification rates. Our analysis (reported in Hobbs et al., 2008 and illustrated in their Figs. 2 and 3) indicates that most of the deformation reported in Figure 3 of Schmid et al. (2010) comprises homogeneous shortening of early formed low amplitude deflections and that buckling plays very little role in producing their "folds". The same results can be read from Figures 2 and 3 of Hobbs et al. (2008) where the dominance of homogeneous shortening is clearly illustrated. We confirm that the velocity arrows shown in Figure 2 of Hobbs et al. (2008) do indeed represent perturbation velocities and not the total velocity (as mistakenly claimed by Schmid et al., 2010) so that the degeneration of the deformation to near homogeneous shortening is clearly illustrated.

## 3. The scale issue

Many processes that operate during deformation of rocks can be expressed as diffusion equations. Thus diffusion of heat, of chemical components and of fluid pressure is governed by equations of the form: $\partial c/\partial t = \kappa^{\text{process}}(\partial^2 c/\partial x^2)$ involving a diffusivity, $\kappa^{\text{process}}$, for the process. $c$ represents temperature, chemical concentration or fluid pressure. If this process is coupled with deformation then the length scale, $l^{\text{process}}$, over which feedback is important is given by the standard diffusion relationship, $l^{\text{process}} = \sqrt{\kappa^{\text{process}}\tau}$ where $\tau$ is a timescale associated with the deformation. We take $\tau = (\dot{\varepsilon})^{-1}$ where $\dot{\varepsilon}$ is the strain-rate; then $l^{\text{process}} = \sqrt{\kappa^{\text{process}}/\dot{\varepsilon}}$. Values of $l^{\text{process}}$ are given in Table 1 which shows that the length scale likely to characterise a particular coupled process varies from kilometres to microns depending on the process and the rate of deformation. We chose thermal–mechanical coupling as a first study in this spectrum of coupled behaviour because it is conceptually the easiest form of coupling to grasp and the computer codes needed to perform the analysis are fully benchmarked (Regenauer-Lieb and Yuen, 2004). For thermal–mechanical coupling the length scale is the kilometre scale. The only point we wanted to make was that regional-scale folds can form through thermal–mechanical coupling. Other scales have been considered in Regenauer-Lieb et al. (2009) and Hobbs et al. (in press) where it is shown that the same fold mechanism (namely, strain-rate softening) operates at outcrop and thin-section scales as does at regional scales although different physical and chemical processes are involved at the different scales.

An issue also involving scale is raised by the "throw away comment" of Schmid et al. (2010) that the second order folds arise from "mesh dependency". Mesh dependency associated with instability of deformation is commonly exhibited by rate independent materials and has been studied for at least 30 years. Schmid et al. (2010) apparently are confusing this kind of behaviour with what occurs in rate dependent materials with strain-rate softening where localisation is not expected to be beset by issues to do with mesh dependency (Needleman, 1988). In our models, in common with other rate sensitive materials, the thickness of the localised zone is scaled by the size of the initial imperfections and not by the mesh. Moreover mesh dependency for this particular problem was studied by Regenauer-Lieb and Yuen (2004) together with the precautions needed to ensure numerical stability. The second order folds in Hobbs et al. (2008) arise from a self-enhancing feedback loop initiated from heterogeneities in thermal expansion, not from mesh sensitivity.

## 4. Concluding remarks

Our paper presents just one of many different theories that have been or might be developed for folding that is different to Biot's approach. We do not disagree with the proposition that some natural folds form by a Biot process. We do not disagree with the proposition that large (>20) viscosity ratios exist in nature, although in the mid- to lower-crust we think this is unlikely. Our point is that there is a way of coupling the interesting processes observed in deformed metamorphic rocks within a unifying framework that is not *ad hoc* and that is compatible with the second law of thermodynamics; this leads to folding (and boudinage) by mechanisms that are non-Biot in character.

In summary:

(i) The Biot theory is a linear theory where the distribution of unstable wavelengths at the moment of instability is represented as a Fourier series in which the growth of each mode is independent of all others.
(ii) At finite deflections, only one wavelength survives; natural folds demonstrating a range of wavelength to thickness ratios

are evidence that other non-linear, non-Biot processes have operated.

(iii) Boundary conditions control the dominant wavelength that grows to finite size. Hence, mechanical properties such as viscosity ratios cannot be inferred from natural fold geometries unless the boundary conditions are known.

(iv) Non-linearities such as geometrical, strain or strain-rate softening can lead to interactions between the growth of unstable modes so that localised fold packets develop. The concept of a dominant wavelength is irrelevant in such situations.

(v) Strain-rate softening induced by thermal–mechanical feedback produces localised folding at the kilometre scale. Identical folds are developed at finer scales depending on the coupled processes that lead to strain-rate softening at that scale.

(vi) The larger the viscosity ratio the greater the viscous dissipation and the larger the feedback effect. High viscosity ratios intensify the effect discussed in Hobbs et al. (2008) rather than detract from it.

(vii) The Biot process undoubtedly occurs in natural deformation but other (non-linear) processes are also important and, we believe, dominate in many metamorphic rocks.

Finally, it is difficult to understand how the theoretical basis developed by Biot can be advocated as a unifying approach to rock deformation when it predicts folds in a shortening layered material with moderate viscosity ratios and power-law stress exponents less than 5 but fails to predict boudinage in identical materials in extension. These same materials, that in addition show strain-rate softening, develop both folding and boudinage instabilities (Hobbs et al., 2009, in press).

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